MATH 90 – CHAPTER 1 Name: .

1.1 Introduction to Algebra

Need To Know

- What are Algebraic Expressions?
- Translating
  - Expressions
  - Equations

What is Algebra?

They say the only thing that stays the same is change. Our physical world is always changing and varying. In order to understand, interpret and predict the physical world we need a math way to express variableness – Algebra.

Algebra revolutionized the way we interact with the world.

Algebra is the power to translate the real world into mathematics.

This course will give the skills to exercise and understand this power.

Expressions

Definitions:

A _____ is a letter used to represent a number that can change or that is unknown.

A _____ is another name for a number.

An _____ is a math statement with variables and/or numbers, often with operations signs and grouping symbols.

Examples: w + 10, $\frac{z}{9}$, 2y(a + 3), 5, x

Evaluate the Expression

Evaluate means find the value.

Evaluate the expressions below:

$13 - z$ when $z = 6$

$\frac{5z}{y}$ *when* $z = 9$ *and* $y = 15$

Translation

2 more than Bill's age.	a + b	Add, sum of, plus, more than, increased by
4 less than d.	a – b	Subtract, difference, minus, less than, decreased by
The sum of 7 and twice n	ab, a•b a(b)	Multiply, product times, twice, of
83% of the possible pts.	a÷b a/b $\frac{a}{b}$	Divide, quotient, ratio of, per

Equations

Definitions:

An _____ is a math sentence that sets _____.

Examples:

2 + 7 = 9

5(4) = 10

x – 3 = 9

Equations

Definitions:

A _____ is a number for the variable that makes the equation _____.

Examples:

Is 7 a solution to 94/y = 12?

Translate:

15% of all waste is recycled. This is the same as 47 million tons of recycled material. What's the total waste generated?

end

1.2 The Laws of Algebra

Need To Know

- Some of the Laws of Algebra
  - Commutative
  - Associative
  - Distributive

Commutative Law

Commutative Law of **Addition**

- _____
- Changing _____ in addition is still equivalent.

Commutative Law of **Multiplication**

- _____
- Changing **order** in multiplication _____.

Associative Law

Associative Law of **Addition**

- _____
- Changing _____ in addition is still equivalent.

Associative Law of **Multiplication**

- _____
- Changing **groups** in multiplication _____

Distributive Law

Distributive Law

- _____
- Multiplication distributes across addition

Examples

Check for Understanding

Match the statement to its corresponding Law

Math Statement	Laws
x(9w) = (x9)w	Commutative
x + 9 + w = 9 + x + w	Associative
4(a – 5) = 4a – 20	Distributive

Check for Understanding

Identify which law(s) correspond to each statement

1. t + (3 + w) = (3 + w) + t

2. (7 + y) + x = 7 + (x + y)

Distributive Law – in reverse

Definition:

Factoring or factor (verb) _____

_____ (noun) parts in a multiplication

_____ parts of an expression separated by + or –

The Distributive Law backwards: ab + ac = a(b + c)

Examples: Factor each expression.

5y + 5z

9 + 9x

14a + 56b + 7

end

1.3 Fractions

Need To Know

- Prime Factoring
- Operations on Fractions
  - Simplify (Reduce)
  - Multiply
  - Divide
  - Add
  - Subtract

Vocabulary

Definitions:

Prime numbers – are numbers that can only be factored by one and itself.

{

Prime factoring - means to write a number as a product of only prime numbers.

Prime Factoring

Prime factor 48

Prime factor 180

Reducing Fractions

Reduce $\frac{9}{21}$

Reduce $\frac{210}{98}$

Multiplication of Fractions

Simplify each expression

$\frac{6}{5}\left(\frac{2}{7}\right)$

$\frac{2}{3}\cdot\frac{5}{x}$

$\frac{9}{2}\cdot\frac{4}{3}$

Recall fraction multiplication

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

Division of Fractions

Recall – Division of fractions is _____

_____.

$$\frac{\mathbf{a}}{\mathbf{b}}\div\frac{\mathbf{c}}{\mathbf{d}}=\frac{\mathbf{a}}{\mathbf{b}}\cdot\frac{\mathbf{d}}{\mathbf{c}}$$

Simplify:

$\frac{7}{9}\div\left(\frac{1}{6}\right)$

$15\div\left(\frac{3}{2}\right)$

Add and Subtract Fractions

Recall the method to add fractions

Recall the method to subtract fractions

Renaming Fractions

Recall how to use the _____ to rename fractions.

$\frac{9}{16} = \frac{\_\_}{48}$ $\frac{19}{42} = \frac{\_\_\_}{210}$

Least Common Denominator

Definition –

The least common denominator (LCD) is the _____

_____

Ways to find the LCD –

1) Use intuition

2) Use the prime factoring method

Find the LCD

$\frac{1}{6}, \frac{1}{10}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$

How to find the LCD

Find the LCD for

1) 12 and 18 2) 60 and 42

Steps to find LCD

0. _____ OR
1. Prime _____ each denominator
2. Create a product
   - using _____ kind of factor
   - raised to the _____ that occurs in any one factoring

Add and Subtract Fractions

Recall how to add fraction

$$\frac{5}{7}+\frac{5}{21}$$

How to add or fraction

Find the LCD

Rename each fraction
(use the "fancy one")

Add numerators

Reduce

Practice

- **How to + or – fraction**

1. Find the LCD
2. Rename (use the "fancy one")
3. Add numerators
4. Reduce

Simplify:

$$\frac{7}{8}-\frac{5}{10}$$

Simplify:

$$3-\frac{3}{5}$$

end

1.4 Subsets of Real Numbers

Need To Know

- Subsets of the Real Numbers
- Comparisons Symbols
- Absolute Value

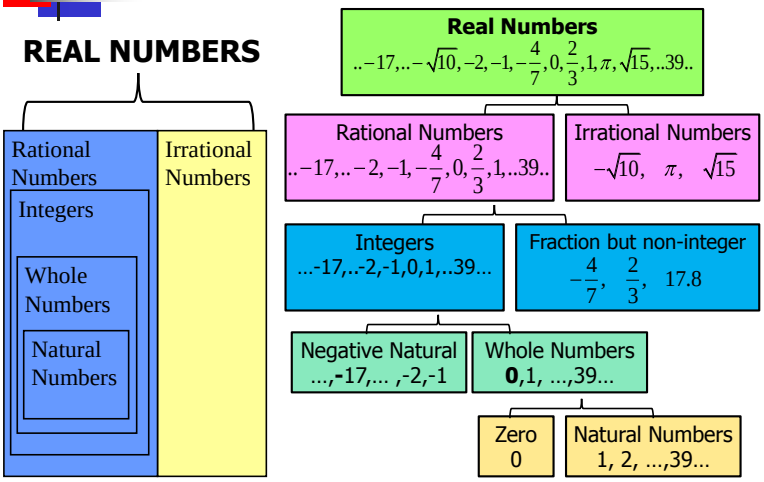
Number Sets

_____ Numbers are all _____ Numbers.

Irrational Numbers are the numbers that aren't Rational.

_____ Numbers are the _____.
(numbers of the form a/b where b is not zero)
_____ are the numbers = { }
Negative Natural Numbers + Whole Numbers
_____ Numbers are the numbers = { }
{0} + Natural Numbers
_____ Numbers are the numbers = { }

Number Sets – Two Diagrams



Check for Understanding

$\left\{-5, -0.25, 0, 1, \pi, \frac{2}{7}, \sqrt{2}, 0.33\overline{3}, 5\right\}$ Categorize and list the numbers from the set to each set below.

Natural Numbers
{ }

Rational Numbers
{ }

Whole Numbers
{ }

Irrational Numbers
{ }

Integers
{ }

Real Numbers
{ }

Comparison Symbols

True or False

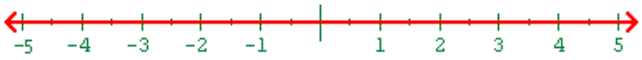
$4 \leq -4$

$-4 \leq 3$

$-4 \leq -4$

$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than **OR** equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than **OR** equal to b

Number Line



Key Vocabulary

Positive Numbers – Numbers to the right of zero.

Negative Numbers – Numbers to the left of zero.

Opposite of a number is on the other side of zero.

Points on the number line correspond to real numbers.
All of the points represent all of the Real Numbers.

Absolute Value – _____

Inequality Comparisons

If a number is further left on the number line,
it is less than (<).

If a number is further right on the number line,
it is greater than (>).

Examples: Fill in the blank with < or >.

-7 __ 3 -¼ __ -¾ |-8| __ |-5|

end

1.5 Addition of Real Numbers

Need To Know

- Two models for addition
- Rules to add signed numbers
- Simplifying Expressions
- Translation

Two Models for Addition

About Addition

1. The _____for adding signed numbers.
2. The _____for adding signed numbers.

Why do we have to look at two models?

- Developing intelligence requires the ability to see things from more than one perspective.
- These models help us generalize the rules of adding.
- In order to really grasp a mathematical concept you need to understand it numerically, analytically and graphically.

Adding Signed Numbers

Financial Model

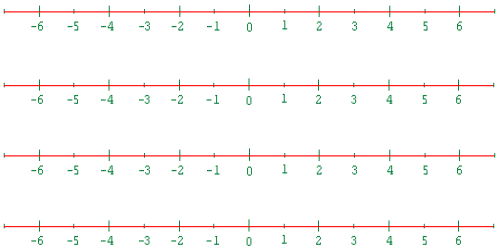
- _____**numbers** correspond to _____or credits to your account.
- _____**numbers** correspond to _____ or debits from your account.

Deposit + Deposit	=
Debt + Debt	=
Deposit + Debt	=
Debt + Deposit	=

Adding Signed Numbers

Vector Model (Adding with the Number Line)

- **Vectors** are graphs of arrows with length and direction.
- **Positive numbers** are arrows to the right.
- **Negative numbers** are arrows to the left.



Examples

1) 2 + 5
2) (-3) + (-1)
3) (-5) + 2
4) 4 + (-6)

Adding Signed Numbers-Rules

Rules for Adding

1) If the signs are the _____, ____ values and _____the sign.

2) If the signs are the _____, _____values and keep the _____ _____ the value.

Examples

1) 4 + 3
2) (-2) + (-6)
3) (-5) + 2
4) 8 + (-5)

Adding Like Terms

- _____are parts of an expression _____ They may be numbers and/or variables often combined with multiplication or division.
- **Numerical Coefficient** is the number factor of a term.
- **Like Terms** are terms with the exact same variable factors.

The Distributive Law helps us simplify expressions:
4x + 9x

Adding Like Terms is as simple as adding coefficients.

-11b + 5b
2x + (-5y) + (-5x) + (-9y)
8 + a +(-5.5a) + 7

Translation and Practice

Write the expression in mathematics and simplify.
The sum of –5 and –11 increased by 4.

Simplify the following expression.
[18 + (-5)] + [9 + (-10)]

1.6 Subtracting Signed Numbers

Need To Know

- Opposites
- Idea of Subtraction
- Rule for Subtraction
- Translation

Opposites

Definition:
The opposite of a number "a" is written ____.

Recall: _____
Example: Find –x and –(-x) when x = 3.

The Law of Opposites
a + (-a) = 0

Idea of Subtraction

Goal: To make a model for distance and
to make the rule of subtraction understandable.

(1)Problem: If a football team makes a play from the 33 yard line to the 39 yard line, how much distance did the team gain.

Idea of Subtraction

Goal: To make a model for distance and
to make the rule of subtraction understandable

(2)Problem: The team punts the football from 2 yards <u>behind</u> the goal line. The ball stops on the 50 yard line. How many yards did the ball travel?

Rule For Subtraction

<u>Rule for subtracting signed numbers</u>

- _____ is the same as _____.
- _____

<u>Examples:</u> Change each to addition

3 – 4
3 – (-4)
-3 – 4
-3 – (-4)

Subtraction of Signed Numbers

Write each as an addition problem and then simplify your answer.

11 – 5

11 – (-5)

-11 – 5

-11 – (-5)

Subtraction Practice

Simplify

3 – 4 – 5

Simplify

-9x + 5 – 3x

24 – (-12) + 7 – 15

-5 + 3b – 7 – 5b

Translation

Difference, decreased, take away, reduced, less and **from** are all key words for subtraction.

Examples: Translate into mathematics and use the rule of subtraction to simplify.

Subtract 5 from 8.

Find the difference of 4 and –7.

1.7 Mult. & Div. of Real Numbers

Need To Know

- Multiplication of Signed Numbers
- Division of Signed Numbers
  - Apply to: Integers, Decimals and Fractions

Sign Patterns in Multiplication

Look at these multiplication problems and draw conclusions about sign results.

(3)(2) =	3(-2) =
(3)(1) =	2(-2) =
(3)(0) =	1(-2) =
(3)(-1) =	0(-2) =
(3)(-2) =	-1(-2) =

Practice - Multiplication

Summary of sign pattern for multiplication

(+)(+) = + (+)(–) = –

(–) (–) = + (–)(+) = –

Simplify each expression

(-4)(-8)(-1) (-3)·(-5)·(-2)·(-1)

The product of an odd # of negatives is _____

The product of an even # of negatives is _____

Practice - Multiplication

Simplify each expression

Recall fraction multiplication

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

$$-9\left(\frac{1}{3}\right)$$

$$-\frac{6}{5}\left(-\frac{2}{7}\right)$$

Division w/ Signs and Fractions

Since $a\div b=a\cdot\frac{1}{b}=\frac{a}{b}$,

the sign rules for DIVISION are the same as for MULTIPLY.

(+) ÷ (+) =
(–) ÷ (–) =
(+) ÷ (–) =
(–) ÷ (+) =

Fraction Facts

Top = _____
Bottom =

$$\frac{0}{5}$$

$$\frac{5}{0}$$

Division of Fractions

Recall – Division of fractions is the same as multiplication by the reciprocal.

$$\frac{\mathbf{a}}{\mathbf{b}}\div\frac{\mathbf{c}}{\mathbf{d}}=\frac{\mathbf{a}}{\mathbf{b}}\cdot\frac{\mathbf{d}}{\mathbf{c}}$$

$$-\frac{7}{9}\div\left(\frac{1}{6}\right)$$

$$-15\div\left(-\frac{3}{2}\right)$$

1.8 Exponents and Order of Op.

Need To Know

- Exponents
- Order of Operations
- Simplifying Expressions

Exponents

Exponents mean repeated multiplication ___

Notation: 4^3

Examples:

5^4

$(2x)^5$

$(-7)^2$

-7^2

Practice - Order of Operation

Order of Operations –

Always work left to right

1. Evaluate _____.
2. Evaluate _____.
3. Evaluate _____ in order _____.
4. Evaluate _____ in order _____.

Simplify:

$20 \div 5 + 15$

$8 \div 2 \cdot 4$

$12 \div (-3 - 5)$

Practice - Order of Operation

Order of Operations –

Always work left to right

1. Evaluate grouped expressions.
2. Evaluate exponents.
3. Evaluate multiplication or division in order left to right.
4. Evaluate addition or subtraction in order left to right.

Simplify:

$$-2(6-10)-3|5-8|$$

Practice - Order of Operation

Order of Operations –

Always work left to right

1. Evaluate grouped expressions.
2. Evaluate exponents.
3. Evaluate multiplication or division in order left to right.
4. Evaluate addition or subtraction in order left to right.

Simplify:

$$-2\cdot 5^2+3\cdot 2^3\div(-1)^4$$

Practice - Order of Operation

Order of Operations –

Always work left to right

1. Evaluate grouped expressions.
2. Evaluate exponents.
3. Evaluate multiplication or division in order left to right.
4. Evaluate addition or subtraction in order left to right.

Simplify:

$$\frac{6(-2)+5(-3)}{5(4)-11}$$

Simplifying Expressions

Recall: -1(a) = -a, and that opposite and negative are synonymous

What is $-(a + b) =$

Examples:

$-(7z + 6)$

$-(13y - 5x + 8)$

$-(-8x^3 + 4x^2 - 3x)$

Simplifying Expressions

Examples:

$7y - (2y + 9)$

$9t - 5r - 2(3r + 6t)$

$8n^2 + n - 7(n + 2n^2)$